The Dynamics of Sovereign Credit Risk

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May 27, 2009

Abstract

This paper provides a model for sovereign default risk valuation in which a sovereign country endogenously determines the timing of default on its external debt. The theoretical relationships between credit spreads and the macroeconomic factors considered in the model are consistent with the empirical literature. The model explains the variation across time in Emerging Market Bond Index (EMBI+) spreads to a degree not offered by existing theoretical and empirical models. I use daily price data information on stock market indices to compute credit spreads for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008. The out-of-sample analysis focuses on the subprime crisis period from January 1, 2007 through December 31, 2008. The model explains 32% of the out-of-sample time variation in EMBI+ spread changes. The enhanced explanatory power of the model is due to a transformation of the non-linear relationship between observed EMBI+ spreads and daily stock market prices into a linear relationship between the model spreads and EMBI+ spreads. The inclusion of additional variables (VIX, S&P 500 return, and U.S. Treasury Rate) raises the explanatory power to 45%. I also show that accounting for a time-varying sovereign’s incentive to default can best explain the level of EMBI+ spreads, especially during during the recent period of distress.

JEL Codes: F34, G12, G13, G15

Keywords: Sovereign Debt, Credit Risk, Sovereign Spreads, Debt Renegotiation

∗Acknowledgements: I am deeply grateful to Darrell Duffie and Bernard Dumas for insightful discussions and priceless comments. This paper has also greatly benefited from suggestions provided by Laura Alfaro, Daniel Andrei, Tim Bollerslev, Ricardo J. Caballero, John Y. Campbell, Jeffrey A. Frankel, Ricardo Hausmann, Michael Hutchison, Jean Imbs, Eric Jondeau, Philippe Jorion, Robert C. Merton, Erwan Morellec, Pascal François, Rajna Gibson, Roberto Rigobon, René Stulz, Michael Waibel, Vivian Yue, participants of the 2007 Paris Finance International Meeting, the 2008 Financial Risks International Forum on Structured Products and Credit Derivatives, the 2008 Derivatives Securities and Risk Management FDIC Conference, the 2008 Swiss Doctoral Workshop in Finance, the 2008 European Finance Association, and seminar participants at Harvard University; UC Santa Cruz, University of Lausanne, and University of Zurich. I acknowledge the financial support from Swiss Finance Institute and the NCCR FINRISK, managed by the Swiss National Science Foundation. Earlier versions of this article have circulated as “A Structural Model for Sovereign Credit Risk” and “Sovereign Credit Risk with Endogenous Default”. This research has been completed while the author was a visiting research fellow at Harvard University and UCLA Anderson. All errors, conclusions and opinions contained herein are solely those of the author.

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1 Introduction

This paper provides a model for evaluating sovereign default risk. The model explains variation across time in Emerging Market Bond Index\(^1\) (EMBI+) spreads better than existing theoretical and empirical models. It also highlights theoretical relationships between credit risk and other macroeconomic variables that are consistent with the existing literature. The importance of correctly evaluating sovereign credit risk is clear, given the important role played by sovereign debt in financial markets. Sovereign foreign debt is currently the largest asset class in emerging markets, representing approximately $5,500 billion of principal in 2007 (FT, 2007). This debt has also been at the center of a number of international lending crises.

I generate estimates of daily credit spreads for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008 and compare these estimates to observed EMBI+ spreads. I rely on price data on country stock market indices, which measure a country’s economic performance, to generate daily credit spreads. Stock market prices exhibit severely non-linear relationship with EMBI+ spreads. The contribution of the paper resides in the development of a structural model that breaks down the non-linearity in this data in order to generate estimates of credit spreads that are linearly related to EMBI+ spreads. A first-passage time model with a constant endogenous default boundary, which captures the sovereign’s incentive to default, characterizes the benchmark framework. I also extend this model to allow for a time-varying incentive to default that depends on both country specific and macro wide economic conditions. I estimate the model parameters over the period from January 1, 1996 to December 31, 2006. The out-of-sample analysis consists of the subprime crisis period from January 1, 2007 through December 31, 2008.

Regarding the results, changes in the estimated credit spreads explain 32% of variation across time in daily EMBI+ spread changes over the out-of-sample period.\(^2\) The explanatory power rises to 35% when accounting for the option-implied volatility index (VIX) as an additional time-varying factor, and to 45% when additionally considering the return on the S&P 500 and the 5 year U.S. Treasury rate. However, the benchmark model is weak in the explanation of the level of observed credit spreads. In contrast, accounting for a conditional incentive to default significantly improves the explanation of the sample characteristics of EMBI+ spreads, in addition to explaining the time-variation. The results hold during both quiet times and during the recent subprime crisis period. Hence, information on both country-specific and market

\(^1\)JPMorgan, one of the major dealers in the Brady market, derives the credit spread implied in the price of each Brady bond. The company computes each country’s EMBI+ index, which is a weighted average index of spreads using the country’s most liquid Brady bonds. A focus on Brady bonds rather than other emerging market instruments is likely to improve the analysis of credit risk as they are by far the most liquid and the largest emerging debt market.

\(^2\)In contrast to the existing empirical literature, this paper only explores the dynamics of credit spread changes. An analysis on credit spread levels would raise severe nonstationarity issues.
wide economic conditions appear to be successful and necessary in the explanation and the prediction of the dynamics of sovereign credit risk. This finding may partially change interpretations of the results of Longstaff et al. (2007) and Pan and Singleton (2008). These authors show that credit risk movements are mostly attributed to changes in common factors across countries.

This paper offers the first structural model that explains the dynamics of daily EMBI+ spreads. Prior studies interested in daily spreads have considered either a reduced-form affine structure model\(^3\) or a reduced-form contingent-claims analysis.\(^4\) Studies analyzing the dynamics of quarterly or annual credit spreads have used a dynamic stochastic equilibrium model\(^5\) and a panel-based approach.\(^6\) An advantage of the proposed model is that it provides an intuitive theoretical framework for the determinants of credit spreads. As such, the model can then be used to motivate further empirical analyses, one of the aims of this paper.

The theoretical relationships between credit risk and other macroeconomic factors analyzed in this model are consistent with the existing literature. First, within the model, credit risk decreases with economic performance because the sovereign country is more likely to default in a recession. This is consistent with previous empirical work.\(^7\) Second, higher macroeconomic volatility leads to greater credit risk, also consistent with prior empirical findings.\(^8\) Third, the model suggests that credit risk increases with the risk-free interest rate, which is observed in the data.\(^9\) Fourth, the model implies that sovereign credit risk is negatively affected by the severity of economic losses upon default, which has been confirmed empirically.\(^10\) That is, a default crisis reduces the sovereign country’s access to the international trade market, thereby reducing future sovereign revenues. If the sovereign country relies heavily on trade, then it is less inclined to default, because the economic impact of defaulting is large.

The model also accounts for endogenous renegotiation upon default. Once default occurs, the sovereign country and its lenders renegotiate the terms of their debt contracts. The importance of incorporating renegotiation upon default in the model is evident by the degree of debt reduction observed during sovereign default episodes (Moody’s, 2006). The outcome of the restructuring process involves a Nash bargaining solution. Within the model presented here, the sovereign country benefits from restructuring by avoiding economic sanctions. Depending on the relative bargaining power of the two parties, the sovereign can either

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\(^3\)Duffie and Singleton (1999), Duffie, Pederson, and Singleton (2003), Longstaff et al. (2007), and Pan and Singleton (2008).
\(^6\)Hilscher and Nosbusch (2008) and the references therein.
have greater or lower incentive to default. In comparison, Yue (2006) obtains that renegotiation necessarily increases credit risk. Finally, I provide intuitive theoretical relationships between the endogenous recovery rate upon default and the macroeconomic fundamentals previously mentioned.

This paper builds on the foundation laid by important prior work. One body of literature, launched by the seminal contributions of Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), addresses why sovereign lending takes place by focusing on the costs of future access to credit, trade, and financial markets, as well as on retaliatory actions by way of sanctions. However, these frameworks do not provide a clear understanding of why or when a sovereign country defaults. In response to this lack of clarity, a second body of study has emerged. Gibson and Sundaresan (2001), Westphalen (2002), and Francois (2006), for example, offer a model for evaluating sovereign debt in the presence of strategic default using a contingent-claims framework. However, these studies do not analyze the dynamics of credit spreads. In contrast, a more recent body of literature (Yue, 2006; Arellano, 2008) draws on dynamic stochastic equilibrium models to explain the dynamics of Argentinean spreads using quarterly GDP data. Finally, Bodie et al. (2007) and Gapen et al. (2008) suggest using higher-frequency forward-looking data on the economy, as provided by market prices. They use domestic debt prices to infer the market value of sovereign wealth, which is then used to derive daily indicators of sovereign credit risk on foreign debt.

The approach considered in this paper merges these four bodies of literature. Default is triggered when the level of the sovereign country’s assets (determined by the present value of future sovereign country’s revenue) falls below an endogenous default boundary dependent on economic fundamentals of the sovereign country. The daily market value of a sovereign country’s assets is proxied using that country’s stock market index, which is a forward-looking measure of future corporate income. I do this, because the sovereign country’s revenue comes almost exclusively from corporate tax income in emerging markets. I thus depart from Merton’s (1974) approach, which was developed for corporate debt valuation. Within my model, equity is not viewed as a liability on the sovereign balance-sheet equal to the residual value of that sovereign country’s assets after the promised payments to debt holders have been made; rather, equity is viewed as a proxy for the market value of the sovereign country’s assets. There are two reasons for this. First, since equity is not a source of sovereign financing, it cannot be a liability. Second, by the very nature of sovereign default, sovereign debt is not an assets claim. Combining an intuitive theoretical model with market valuation, this paper provides a framework that determines the timing of sovereign default and, thus, the price of sovereign debt.

The rest of the paper is organized as follows. Section 2 outlines a theoretical understanding of endogenous
default policy for a sovereign country. In Section 3, I offer some of the model’s theoretical predictions for the relationships between sovereign credit risk, the recovery rate upon default, and its determinants. The model is then applied to EMBI+ spread data in Section 4. I conclude my analysis in Section 5.

2 The Model

Throughout the analysis, I assume that capital markets are frictionless and that all investors have perfect information on the state of the economy. All variables are measured in the currency of the lenders. The default-free term structure is flat with an instantaneous riskless rate $r$, at which investors may lend and borrow freely.

I consider a sovereign country with an infinite lifespan. The country is constituted of a representative unlevered firm, which generates a stream of operating income denoted by $X$. Corporate taxes are paid to the sovereign at a rate $\tau$ on operating cash-flows. The value $V$ of the sovereign country’s assets is determined by the unlimited value of a perpetual claim to the current taxed operating income, which I define by $V_t = \mathbb{E}_Q[\int_t^{\infty} \tau X_u e^{-ru} du]$. The firm’s equity is a tradable asset and its price is defined by $S_t = \mathbb{E}_Q[\int_t^{\infty} (1-\tau)X_u e^{-ru} du]$. As $V_t = \frac{\tau}{1-\tau} S_t$, the assets of the sovereign country can be synthetically replicated by investing in the firm’s equity share. The sovereign country’s assets are then indirectly tradable, and the markets are complete.\(^{11}\) The value of the sovereign country’s assets $(V_t)_{t \geq 0}$ is governed under the risk-neutral measure $Q$ by the process

$$dV_t = rV_t dt + \sigma V_t dZ^Q_t, \quad V_0 > 0, \quad (1)$$

where the process $Z^Q_t$ is a Brownian motion defined on the probability space $(\Omega, \mathcal{F}, Q)$. The standard filtration of $Z^Q_t$ is $F = \{\mathcal{F}_t : t \geq 0\}$. The volatility of the diffusion process is given by $\sigma$.

The sovereign country can choose to default on its debt obligations. According to standard default clauses in sovereign debt contracts, the sovereign loan is assumed to be in default when the sovereign country fails to pay the debt service on schedule.\(^{12}\) Most defaults occur during economic downturns (Reinhart et al., 2003 and De Paoli et al., 2006). As such, a sovereign country may strategically default on its debt obligation when its assets fall below a level $V_B < V_0$. Reorganization of the debt contract with lenders is initiated at

\(^{11}\)In Merton (1974), the dynamics of the firm’s assets is unobservable and inferred from stock market information. Instead, I view the sovereign country’s assets as observable and tradable.

\(^{12}\)Default clauses contain more details and allow for other sources of default. For instance, a grace period of 30 days is generally considered. Leaving the IMF as a member country is legally considered as defaulting on the debt. Nevertheless, the simple definition adopted here also corresponds to that of Standard and Poor’s.
\[ T(V^B) = \inf \{ t \geq 0 \mid V_t \leq V^B \}. \]

I first evaluate the price of sovereign debt for a given default boundary \( V^B \) and then determine the selected threshold endogenously.

### 2.1 The Price of Sovereign Debt

I assume an infinite maturity debt contract,\(^1\) characterized by a level \( D \) and a continuous debt service \( c \) until default. The pricing formula of sovereign debt \( D \) does not require knowledge of either investor tastes or beliefs about the expected growth rate of the sovereign country’s assets. Furthermore, the pricing formula must avoid arbitrage opportunities. That is, from a standpoint of market completeness, a portfolio strategy that holds underlying assets \( V \), debt \( D \), and risk-free assets can be modified to be riskless and thus avoid arbitrage by picking appropriate weights such that the expected return on the portfolio equals the risk-free rate \( r \). Using Itô’s lemma, the value of the perpetual debt satisfies

\[
rd = c + rDV + \frac{1}{2} \sigma^2 V^2DVV 
\]

where \( D_V \) and \( DVV \) are the first and second derivatives, respectively, of the sovereign debt value \( D \) with respect to the sovereign country’s assets \( V \). The solution to this ordinary differential equation is subject to a number of conditions. First, when the sovereign country’s assets \( V \) tend to infinity, the value of the sovereign debt tends to the value of risk-free debt

\[
\text{Lim}_{V \to \infty} D(V) = \mathbb{E}_Q \left[ \int_0^\infty ce^{-rt} dt \right] = \frac{c}{r}
\]

Second, lenders must evaluate the foreign debt upon default; this evaluation depends on the recovery rate. That is, upon default, the sovereign country and its lenders restructure the terms of the debt contract and agree on a reduction in the debt service. I determine the value-matching conditions that impose equality between the value of the sovereign debt and the value of the restructured debt upon default. At default time \( T(V^B) \), the value of the sovereign debt is

\[
\text{Lim}_{V \to V^B} D(V) = \frac{c(1 - \phi)}{r}
\]

where \( 0 \leq 1 - \phi \leq 1 \) denotes the recovery rate on the debt service \( c \). I assume that the sovereign country...
cannot increase its debt after default.

The value of sovereign debt is associated with the relevant boundary conditions (Eqs. 3 & 4):

\[
D(V) = \mathbb{E}_Q \left[ \int_0^{T(V_B)} c e^{-rt} dt \right] + \mathbb{E}_Q \left[ \int_{T(V_B)}^{\infty} (1 - \phi)c e^{-rt} dt \right] 
\]

\[
= \frac{c}{r} \left[ 1 - \left( \frac{V}{V_B} \right)^\beta \right] + \frac{c (1 - \phi)}{r} \left( \frac{V}{V_B} \right)^\beta \quad (5)
\]

\[
= \frac{c}{r} \left[ 1 - \phi \left( \frac{V}{V_B} \right)^\beta \right] 
\]

\[
= \frac{c}{r} \left[ 1 - \phi \right] \left( \frac{V}{V_B} \right)^\beta \quad (6)
\]

\[
= \frac{c}{r} \left[ 1 - \phi \right] \left( \frac{V}{V_B} \right)^\beta \quad (7)
\]

where \( \beta \) is the negative root of the quadratic equation \( \frac{1}{2} \sigma^2 \beta^2 (\beta - 1) + r \beta - r = 0 \) that is

\[
\beta = \frac{1}{2} - \frac{r}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{r}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0 \quad (8)
\]

The market value of sovereign debt \( D(V) \) is equal to a riskless perpetual debt with continuous coupon \( c \) minus a default premium. This premium corresponds to the present value of the unrecovered value of the debt after default, where the stochastic discount factor is defined as the Arrow-Debreu price of default \( \mathbb{E}_Q^{V_B} \left[ e^{-rT(V_B)} \right] = \left( \frac{V}{V_B} \right)^\beta \). Lenders anticipate the opportunistic behavior of the sovereign country by reflecting the associated wealth extraction in the pricing of sovereign debt.

### 2.2 Optimal Default Boundary and Credit Spread

Defaulting incurs economic costs that may occur upon default. First, debt repudiation can impede the ability of the defaulting sovereign country to trade, as empirically documented by Rose (2005).\(^\text{15}\) This is pertinent, since Frankel and Romer (1999) show that trade is a significant part of economic growth. Second, default weakens the domestic financial system. As major creditors of the government, domestic banks may be prevented from competing their intermediary duties of providing liquidity and credit to the economy.\(^\text{16}\) De Paoli, Hoggarth, and Saporta (2006) show that the recent major default crises have been associated with banking crises, which often result in severe and prolonged recessions. More generally, Reinhart et al. (2003) and Sturzenegger and Zettelmeyer (2006) provide ample evidence that the costs of defaulting on external debt can be significant for economic activity. I assume that in the event of default, the sovereign country

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\(^{15}\)As argued by Rose (2005) and Martinez and Sandleris (2008), declines in trade should not be interpreted as trade sanctions imposed by creditor countries. Lenders are indifferent to the trade access of the defaulting sovereign as they are not directly involved in the trade market. Among the most important lenders, we generally find the IMF, the World Bank, and various commercial banks (De Paoli et al., 2006).

\(^{16}\)For instance, Argentina experienced a decrease of bank credit to the private sector (as a proportion of annual nominal GDP) from 20.8% at the end of 2001 (start of the default crisis) to 10.8% at the end of 2003 (De Paoli et al., 2006).
incurs a cost equal to a fraction $0 < \lambda < 1$ of the level of the sovereign country’s assets upon default. Within the model, avoiding this default cost is the sovereign country’s motivation in avoiding default.\footnote{Bulow and Rogoff (1989) suggest two additional reasons for a country to repay its foreign debt. First, lenders may be able to appropriate collateral. However, in the event of repudiation, the assets accessible to creditors are worth only a small fraction of the outstanding level of debt (Bulow and Rogoff, 1989). Second, they consider a reputation effect, which will impact future borrowing opportunities, though empirical support for such an effect is weak (Eichengreen, 1987). Gelos, Sahay, and Sandleris (2004) provide evidence that sovereign countries are able to regain market access as soon as three and a half months, on average, after defaulting.}

For given debt service $c$, recovery rate $1 - \phi$, and default boundary $V^B$, sovereign wealth $W(V)$ equals

$$W(V) = \mathbb{E}_Q \left[ \int_0^\infty (V_t - c) e^{-rt} dt \right] + \mathbb{E}_Q \left[ \int_{T(V^B)}^\infty (c\phi - \lambda V_t) e^{-rt} dt \right]$$

(9)

$$= V - \lambda V^B \left( \frac{V}{V^B} \right)^\beta - \frac{c}{r} \left[ 1 - \phi \left( \frac{V}{V^B} \right)^\beta \right]$$

(10)

The first term on the right-hand side of Eq. (9) is sovereign wealth in the absence of default. The second term is the difference between the reduction in debt service and the loss upon default multiplied by the Arrow-Debreu price of this event. The sovereign country’s default policy is characterized by the default boundary $V^B$. It is chosen to maximize sovereign wealth $W(V)$,\footnote{Prior studies have not settled the debate on what exactly the objective function of the sovereign country should be. Here, $W(V)$ corresponds to the part of the economy controlled by the sovereign country on behalf of its citizens, as lenders are foreign investors by assumption. Gibson and Sunderasau (2001) consider that the sovereign country has an objective function that is linear in total country wealth $W(V) + D(V)$. In Westphalen (2002) and Bulow and Rogoff (1989), the sovereign country seeks to maximize terminal total country wealth.} such that the smooth-pasting condition \usepackage{amsmath}

$$\frac{\partial W(V)}{\partial V} |_{V = V^B} = 1 - \lambda$$

is satisfied:

$$V^B = \frac{c\phi}{r\lambda} \left( \frac{\beta}{\beta - 1} \right)$$

(11)

The incentive to default early, as expressed in the model by increasing $V^B$, is to lower the expected value of debt repayment. However, debt holders take this opportunistic behavior into account in the market valuation of debt. On one hand, for a given debt service $c$, the market value of debt $D(V)$ decreases with $V^B$. On the other hand, the expected loss upon default also rises with the incentive of defaulting. The sovereign country must consider this trade-off when determining its default policy.

Under the risk-neutral measure, the sovereign country’s assets grow at the risk-free rate, whereas the default boundary remains constant. The probability of defaulting thus vanishes over time, which may not be a desirable property. Instead, under the assumption that the sovereign continuously issues new debt with coupon $c_t = c_0 e^{rt}$, the default boundary grows at the same rate as the assets. Mathematically, that case corresponds to a stationary environment with a fixed debt service, such that $\beta$ simplifies to $\frac{1}{2} \sqrt{\frac{1}{4} + \frac{2\sigma^2}{\sigma^2}}$. I will further refer to this model as the model with exponential default boundary, in contrast to the model with constant boundary.
The market yield spread is a measure of the market’s perception of default risk. Under the risk-neutral measure, the credit spread is obtained for two fundamental reasons. First, there is a risk of default. Second, in the event of default, the lenders receive only a fraction of the promised payments. The credit spread is defined as

\[
CS(V) = \frac{c}{D(V)} - r
\]

where \( c \) is the promised payment, \( D(V) \) is the defaulted debt, and \( r \) is the risk-free rate. This is known as Equation (12).

\[
= r \left[ 1 - \phi \left( \frac{V}{V^B} \right)^{\beta} \right] - r
\]

is known as Equation (13).

### 2.3 Modeling with Endogenous Recovery Rate

Yue (2006) shows that accounting for endogenous renegotiation upon default between the sovereign country and its lenders helps to (partially) explain the high level of credit spreads. The sovereign country benefits from restructuring debt by avoiding economic sanctions. It has greater incentive to default, which thereby increases credit risk. I then extend the model in order to analyze how debt renegotiation affects a country’s creditworthiness.

I first make a few additional assumptions. When default occurs, lenders take collective action and renegotiate with the sovereign country regarding debt reduction at no cost. Essentially, the renegotiated debt can be considered as a value-redistribution between the sovereign country and the foreign debt holders. The rule of sharing the sovereign country’s assets upon default results from a Nash bargaining game, as proposed by Fan and Sundaresan (2000) and François and Morellec (2004) with regard to corporations and Gibson and Sundaresan (2001), François (2006), and Yue (2006) with regard to sovereign countries. Lenders anticipate the potential for renegotiation, which affects the price of sovereign debt at issuance.

The basic logic behind the renegotiation process is presented here; see the Appendix for a full derivation. Should the debt agreement fail, lenders lose their initial investment such that \( D(V) \mid_{V=V^B, \phi=1} = 0 \). In response, the sovereign country faces direct sanctions upon default that would increase the loss on its assets by a factor, \( k \). However, by renegotiating debt reduction, the inefficient sanctions can be lifted, and the lenders can recover part of the defaulted debt, with \( D(V) \mid_{V=V^B} = \frac{c(1-\phi)}{r} \). Thus, both parties have an incentive to renegotiate the terms of the debt contract. The endogenous recovery rate \( 1 - \phi^* \) of the debt service satisfies

\[
1 - \phi_i^* = \frac{V^B_i (k-1)r (1-\eta) \lambda}{c}
\]

Equation (14).
where $0 < \eta < 1$ is the sovereign bargaining power in the renegotiation game.

3 Comparative Statics with Macroeconomic Factors

The pricing models developed above can be used with scenario and simulation analysis to evaluate the effect of shocks and policies on credit risk. This section explores the sensitivity of sovereign credit spreads with respect to macroeconomic variables (Figure 1). I provide predictions on credit spreads assuming both a constant and an exponential default boundary. As Figure 1 suggests, the qualitative predictions are identical under both assumptions. The parameter values, presented in Table I, reflect the situation of Brazil, Mexico, Peru, and Russia over the 1998 - 2008 period (sources are cited in the last column of Table I).

Table I [about here]

3.1 Credit Spreads in the Benchmark Model

The Figure 1 (upper-left panel) shows that the estimated credit spread decreases with the level of the sovereign country’s assets $V$. Empirical analyses obtain a negative effect of economic performance on credit spreads, confirming the model prediction. The model generates countercyclical bond spreads, in line with the theoretical predictions of Yue (2006) and Arellano (2008). In contrast, Kehoe and Levine (1993), Kocherlakota (1996), and Alvarez and Jermann (2000) offer models in which default incentives are higher in periods of high output, at odds with the empirical evidence that I have cited. As documented empirically by numerous studies, the credit spread increases when the sovereign issues a greater level of debt, which is captured by an increase in debt service $c$.

Figure 1 [about here]

Greater volatility $\sigma$ in the sovereign country’s assets increases the probability of default, thereby raising the estimated credit spread (Figure 1, upper-right panel). Westphalen (2001) shows that changes in stock market volatility, indicating changes in the volatility of the market’s perception of the economy, have a significant and positive effect on credit spreads. Catao and Sutton (2002) and Catao and Kapur (2004) confirm the positive effect of policy induced volatility (fiscal, monetary, and exchange rate policy) on sovereign credit risk. The estimated credit spread increases with the risk-free interest rate $r$ (Figure 1, middle-left panel), which is consistent with the empirical evidence documented by Catao and Sutton (2002) and Catao and Packer (1996), Haque et al. (1998), Monfort and Mulder (2000), Hu et al. (2002), and Catao and Sutton (2002).
and Kapur (2004) for credit spreads and by Haque et al. (1998), and Monfort and Mulder (2000) for credit ratings. This effect is in contrast with the negative sign obtained in Gibson and Sundaresan (2001) and Westphalen (2001).

Finally, the presence of a potential loss $\lambda$ in the sovereign country’s assets partially insures lenders against default. The higher the expected loss upon default, the less the sovereign country is inclined to default. The estimated credit spread then decreases with the expected costs of sovereign default (Figure 1, middle-right panel). As documented by Rose (2005) and Martinez and Sandleris (2008), the reduction in trading activity is an important source of economic costs upon default. If a sovereign country relies heavily on the export market for its revenues, the potential loss upon default is large, thereby lowering the incentive to default. This suggests a negative relationship between the level of exports and credit risk. This prediction is in line with Reinhart et al. (2003), who show a strong negative correlation between the exports-to-external debt ratio and credit risk. Similarly, Ades et al. (2000) and Rowland and Torres (2004) provide empirical evidence that the exports-to-GDP ratio negatively affects sovereign spreads in emerging market economies.

### 3.2 Renegotiation, Credit Spreads, and Recovery Rate

I now investigate how accounting for the renegotiation process influences credit risk. As Figure 1 suggests, the qualitative predictions are very similar to those of the benchmark model. The sovereign country does not necessarily exhibit greater incentives to default when the renegotiation outcome is endogenous. The potential for debt restructuring provides lenders with valuable protection against loss only for some specific parameter values. In particular, the generation of credit spreads higher than the benchmark credit spreads necessitates levels of sovereign bargaining power upon renegotiation $\eta$ above 90%. Thus, in contrast to Yue (2006), accounting for renegotiation upon default may not necessarily generate higher levels of credit spread.

I also explore the relationship between the obtained recovery rate and the model’s macroeconomic parameters. As illustrated in Figure 2, the theoretical recovery rate decreases with macro-volatility $\sigma$ and risk-free interest rate $r$. The model also suggests that the greater the sovereign bargaining power upon renegotiation $\eta$, the lower the recovery rate is upon default. When powerful countries such as Argentina (2002) and Russia (1998) defaulted on their debt, investors indeed experienced relatively low recovery rates (Moody’s, 2006). When varying the base case parameters, the recovery rate predicted by the model ranges between 10% and 40%, which is in line with the data. In Pan and Singleton (2008), surveyed credit default
swap traders in the sovereign debt market assumed an expected recovery rate of 25%. Since 1983, Moody’s (2006) has reported a value-weighted recovery rate on defaulted sovereign issuers of 33\%.

Figure 2 [about here]

4 Time-Variation of Credit Spreads

This section shows that the pricing model developed in this paper helps to explain the time dynamics of credit spreads in emerging debt markets. I generate estimated credit spread prices for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008 and compare these estimates with observed EMBI+ spreads. These four countries are chosen because of their different geopolitical characteristics and levels of credit risk. I first analyze the period between January 1, 1996 and December 31, 2006 and then carry out an out-of-sample study of the subprime crisis, which consists of the period from January 1, 2007 through December 31, 2008. Figure 3 illustrates the tendency for sovereign credit risk to rise dramatically when the economy performs poorly, as measured with daily stock market prices. This relationship is also highly non-linear (see Figure 4). The success of the model will be evaluated by its ability to transform the non-linear relationship between economic performance and the observed credit spreads into a linear relationship between the estimated credit spreads and EMBI+ spreads.

Figures 3 & 4 [about here]

Data for this section are taken from Datastream for IFCG stock market indices (measured in U.S. Dollars), the VIX index, and 5Y U.S. Treasuries rate series, the OECD for GDP data, and from Bloomberg for the EMBI+ spreads. All series consist of 2870 daily observations from January 1, 1998 to December 31, 2008.

4.1 Methodology

As noted in Section 2, the level of the sovereign country’s assets $V$ can be written as a function of the firm’s equity price $S$, with $V_{i,t} = \frac{\tau}{1-\tau} S_{i,t}$. This linear relationship calls for the consideration of the country’s daily stock market index price $S_{i,t}$ as a proxy for the (unobservable) daily value of that sovereign country’s assets $V_{i,t}$. Normalizing the sovereign country’s assets to unity, the computed stochastic discount factor is

$$
\left( \frac{V_{i,t}}{V_{i,t}^{\beta_t}} \right)^\beta = \left( \frac{S_{i,t}}{V_{i,t}^{\beta_t}} \right)^\beta = \left( \frac{S_{i,t}}{V_{i,t}^{\beta_t}} \right) \frac{r(\beta_t-1)S_{i,t}/S_{i,0}}{\phi_i \beta_t} \right)^\beta.
$$

Thus, the credit spread estimated by the model for

\[20] The trading prices on the sovereign country’s bonds thirty days after default were 65% of par for Pakistan (1998), 18% for Russia (1998), 44% for Ecuador (1999), 18% for Ivory Coast (2000), 69% for Ukraine (2000), 33% for Argentina (2001), 66% for Moldova (2001), 66% for Uruguay (2003), 65% for Grenada (2004), and 92% for Dominican Republic (2005).
country $i$ at day $t$ is determined by

\[
CS_{Model,i,t} = \frac{c_i}{D(V_{i,t}, c_i)} - r = r \left[1 - \phi_i \left(\frac{V_{i,t}}{V_{i,0}}\right)^\beta\right]^{-1} - r 
\]

(15)

\[
= r \left[1 - \phi_i \left(\frac{\lambda}{c_i} \left(\frac{r (\beta_i - 1) S_{i,t}}{\phi_i \beta_i S_{i,0}}\right)^\beta\right]^{-1} - r 
\]

(16)

where

\[
\beta_i = \frac{1}{2} - \frac{r}{\sigma_i^2} - \sqrt{\left(\frac{1}{2} - \frac{r}{\sigma_i^2}\right)^2 + \frac{2r}{\sigma_i^2}} < 0
\]

(17)

The parameter values only account for information made available during the in-sample period. The risk-free rate $r$ is determined by the average 5-year U.S. Treasury rate (see Table I). In contrast, the recovery rate $1 - \phi$, the expected level of haircut $\phi c$, the cost of defaulting $\lambda$, and the expected volatility parameter $\sigma$ are unobservable input. I estimate these parameters by minimizing the tracking error volatility between the changes in estimated credit spreads and the changes in EMBI+ spreads over the period 1998 - 2006.

These parameters are determined as follows

\[
\left\{\left(\frac{c}{\lambda}\right)^*, \phi^*_i, \sigma^*_i\right\} = \arg \min_{(\frac{c}{\lambda}), \phi_i, \sigma_i} \sum_t (\Delta CS_{EMBI,i,t} - \Delta CS_{Model,i,t})^2 \quad s.t. \quad 0 < \phi_i < 1
\]

(18)

where $CS_{EMBI}$ and $CS_{Model}$ stand for the observed EMBI+ spreads and the credit spreads estimated by the model, respectively, for country $i$ at time $t$. I report the estimated values in Table II. The indicator $(\frac{c}{\lambda})_t$ captures the sovereign country’s incentive to default: it increases with the quantity of sovereign debt to be repaid and decreases with the economic costs of defaulting. As it can be expected, the incentive to default is found to be more pronounced for the two defaulters, namely Brazil and Russia, than for Mexico and Peru. The recovery rates obtained from this estimation, with an average recovery rate of 36%, are also consistent with those observed in the data. Moody’s (2006) reports a value-weighted recovery rate on defaulted sovereign issuers of 33%. In particular for Russia, the average recovery rate on government bonds was 18% in 1998, which is equal to the recovery rate implied by the estimated model. As such, the model is in line with the data.

Table II [about here]

\[\text{This ratio is particularly large for Russia. The high economic uncertainty this country has experienced over the 1998 - 2000 period is reflected in the high volatility estimate. From Eq. (11), for a given incentive to default, the higher the economic volatility, the lower the endogenous default boundary of the model. Despite the prevailing great uncertainty, the market has priced a relatively high default boundary as credit spreads were traded between 50 and 70% in August 1998. Thus, according to the model, the credit market has anticipated a very high incentive of the Russian government to default on its debt over this period.}\]
4.2 EMBI+ Spreads versus Estimated Credit Spreads

4.2.1 Descriptive Analysis

I compare the estimated credit spreads and EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the in-sample period. A first analysis consists of analyzing the Pearson correlation coefficient between these two series. The results in Table III exhibit a very high correlation between the daily estimated credit spreads and daily EMBI+ spreads. The average correlation is 0.90 in levels and 0.46 in changes. In comparison, the average correlation between the stock market index prices and EMBI+ spreads is −0.30 in levels and −0.09 in changes. Hence, the pricing formula developed in this paper significantly contributes to the enhancement of the relation between EMBI+ spreads and information on stock market data. Figure 5 displays the time-varying correlation between changes in the estimated spreads and changes in observed EMBI+ spreads, as computed using the Dynamic Conditional Correlation model developed by Engle and Sheppard (2001). The correlation is quite stable over time, suggesting that the results are not specific to a particular sub-period.

Table III

In Figure 6, I compare the daily dynamics of the credit spreads estimated from the model with the dynamics of the observed EMBI+ spreads for the period 1998 - 2006. The model credit spreads and the observed EMBI+ spread series evolve very similarly over both quiet and crisis periods: the model captures the substantial and steady narrowing of sovereign spreads in emerging debt markets over the 2004 - 2006 period as well as the rise in credit risk during the Russian and the Brazilian crises in 1998 - 1999 and 2002 - 2003, respectively. Finally, changes in the estimated credit spreads and the EMBI+ spreads exhibit similar volatility clustering (see Figure 7.)

Figures 5, 6, and 7

4.2.2 Econometric Analysis

I now provide an econometric examination of the ability of the estimated credit spreads to explain the dynamics of the EMBI+ spreads. Instead of single time-series estimations, I rely on a panel data analysis to evaluate the quality of the pricing model using a single measure, namely the overall $R^2$. To avoid any nonstationarity concerns, the investigation considers changes in the estimated and observed credit spread

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22To remove the serial correlation on spreads, I first consider the residuals of an AR(3) on both series. I then smooth the conditional correlation with an MA(30) process for a better clarity.
using the following specification

\[ \Delta CS_{EMBI,i,t} = \delta_i \Delta CS_{Model,i,t} + \omega_i + \nu_{i,t} \]  

where I account for country-specific coefficients of elasticity \( \delta_i \) to allow for heterogeneous relations across countries. In addition, \( \omega_i \) characterizes country-specific effects and \( \nu_{i,t} \) is the error term. I correct the heteroskedasticity consistent standard errors to account for temporal and cross-sectional dependence in the panel data using Driscoll and Kraay’s (1998) extension of Newey and West’s non-parametric variance covariance estimator.\(^{23}\) Finally, regressions are estimated with a Random Effect model.\(^{24}\)

If the model is correct, the estimates of \( \delta_i \) obtained from the above regression should be equal to unity. As displayed in Table IV, these estimates are very close to one. More importantly, we cannot reject the null hypothesis that \( \delta_i = 1 \). The second result that Table IV exhibits is that changes in the estimated credit spread can significantly explain the time variation in EMBI+ spread changes (see Analyses 1a, 1b, and 1c). The explanatory power corresponds to 17%. This result is in line with that of the corporate literature. In comparison, using much more variables, Collin-Dufresne, Goldstein, and Martin (2001) show that the factors that should, in theory, determine corporate credit spread changes have an explanatory power of 25%.

Table IV [about here]

### 4.2.3 Global Economic Factors

The results reported in Table IV suggest that country-specific market price data capture a significant fraction of the time variation in EMBI+ credit spread changes. In contrast, Weigel and Gemmill (2006) and Longstaff et al. (2008) show empirical evidence that variations in sovereign credit risk can be mostly attributed to changes in common factors across countries rather than to country-specific fundamentals. The option-implied volatility index (VIX), which captures one source of risk premium in the U.S. equity market, has recently attracted much interest as a possible determinant of these common factors. Pan and Singleton (2008) view the VIX as a central factor in investors’ appetite for exposure to high-yield credit bonds. Longstaff et al. (2008) and Pan and Singleton (2008) show that the VIX is the key factor in explaining

\(^{23}Economic and financial integration imply substantial interdependencies between emerging market economies. As a result, cross-sectional dependence may arise due to the presence of common shocks and unobserved components that become part of the disturbance term. If common factors are unobserved (and uncorrelated with the included regressors), the standard Fixed and Random Effects estimators are consistent. However, the estimated standard errors are biased.

\(^{24}The choice of the appropriate model is determined here using the Hausman specification test. This test compares the Fixed versus Random Effects under the null hypothesis that the individual effects are uncorrelated with the other regressors in the model. Within this specification, the individual effects are uncorrelated (and thus \( H_0 \) is not rejected). A Fixed Effect model would produce biased estimators, violating one of the Gauss-Markov assumptions, so a Random Effect model is preferred here.
sovereign credit risk movements.

Consistent with these studies, the average correlation between the level of the VIX and the level of EMBI+ spreads, using the data set of this paper, is 29%. Figure 5 shows the conditional correlation between EMBI+ spreads and the VIX for the four countries. This correlation is very similar across countries and appears to be important, although generally lower than the correlation between the EMBI+ spreads and the estimated credit spreads. Following the aforementioned studies, I consider the VIX to be an additional explanatory variable. The results suggest that the VIX has only limited scope to explain additional time variation in the daily EMBI+ spreads (see Table IV, Analysis 1b), although its coefficient is statistically significant. The inclusion of the VIX merely raises the explanatory power from 17% to 18%.

I also consider additional global factors that are likely to explain the time variation in the EMBI+ spreads. The first explanatory variable is the daily 5-year U.S. Treasury rate. The second variable is the daily return on the S&P 500 index. The results in Table IV show that the U.S Treasury rate and the conditional stock market return in the U.S. improve the quality of the estimation only slightly (see Analyses 1b and 1c). The analysis thus suggests that, over this sample, most of the information contained in these common factors (VIX, return on S&P 500, and U.S. Treasury rates) are already factored in the country-specific stock market prices.

### 4.2.4 Credit Spreads Estimated with Quarterly Accounting Data

Instead of considering market prices, Yue (2006) and Arellano (2008) analyze the dynamics of quarterly sovereign credit spreads using information provided by quarterly GDP data in a stochastic general equilibrium model. Yue (2006) explains 78% of the time variation in quarterly EMBI+ spreads for Argentina over the period 1994 - 2001. In another study, Hilscher and Nosbusch (2008) explain 48% of the time variation in EMBI+ spreads over the period 1993 - 2004 using both reduced-form and simple structural models. Hilscher and Nosbusch (2008) use the terms of trade to proxy the state of the economy, rather than GDP data. In contrast to the present paper, this literature focuses on the time-variation in the level of sovereign spreads. However, both the credit spread and the GDP series are typically nonstationary, which can raise severe nonstationarity issues. Hence, it would not be appropriate to compare the econometric results presented so far with those obtained in these studies.

Nevertheless, I here follow Yue (2006) and Arellano (2008) in the computation of the estimated credit spread using quarterly GDP data, a proxy for the sovereign country’s assets. In Figure 8, I compare the

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25I have also considered the effect of the sovereign country’s stock market volatility, computed as the conditional volatility of the stock market log returns using an AR(3)-GARCH(1,1) process. This variable was not found to improve the results and has then been dropped for space consideration.
dynamics of the credit spreads estimated by the model with those of the EMBI+ spreads for the period 1998 - 2006. The estimated model parameters are presented in Table II. The model and observed spread series evolve very closely over time. Figure 9 plots the changes of the estimated credit spreads and the EMBI+ spreads. Again, these changes are very similar. The average correlation between these two series is 72% in levels and 60% in changes (Table III), which is consistent with Yue (2006) and Arellano (2008). Then, the estimated credit spreads based on GDP data seem to explain a substantial part of the time variation in quarterly EMBI+ spreads. However, these data are only available quarterly and, moreover, are reported with a lag. They also provide an ex-post measure of economic performance, whereas sovereign credit spread is a forward-looking market measure. In light of this, stock market prices certainly constitute a more useful determinant for estimating credit spreads. First, investors’ opinions usually reflect the best available, forward-looking data on the economic prospects of a given country. Second, market prices are available at a higher frequency. Third, it appears important to incorporate sentiments on the economy as reflected in stock market prices in the estimation of sovereign credit risk.

Figures 8 & 9 [about here]

4.3 Time-Varying Incentive to Default

A common feature of the existing sovereign credit risk literature is the assumption of a constant default boundary. This section now departs from this hypothesis and suggests to consider a dynamic sovereign country’s incentive to default that is affected by economic conditions. This channel has been recently proposed by Chen et al. (2008) to explain corporate spreads in the U.S. I now define the sovereign country’s incentive to default \( \xi \) as a function of both country specific and market wide economic conditions, which reconciles the results highlighted in Section 3 and those of Longstaff et al. (2008) and Pan and Singleton (2008). To do so, I account for information on the corresponding country’s stock market price index \( S_i \) and the option-implied volatility index \( VIX \), respectively.

I first derive for each country \( i \) and day \( t \) the sovereign country’s incentive to default implied by the level of EMBI+ spreads through an inversion of Eq. (15). Thus, the implied incentive to default explains by construction the level of EMBI+ spreads at any point in time. The implied incentive to default, denoted by \( \xi^{implied} \), satisfies

\[
\xi^{implied}_{i,t} = V_{i,t} \left( \frac{\beta_i - 1}{\beta_i} \right)^{1/\phi_i} \left( \frac{CS_{EMBI,i,t}}{CS_{EMBI,i,t} + r} \right)^{-1/\phi_i} 
\]
where, for each country \( i \), the constant country-specific volatility parameter \( \sigma_i \) is computed as the unconditional volatility of the daily log returns on the corresponding stock market index over the period 1998 - 2006. The recovery rates \( 1 - \phi_i \) are those estimated in Section 4.1 and presented in Table II. The Figure 10 plots the dynamics of the implied incentive to default for Brazil, Mexico, Peru, and Russia over the period 1998 - 2006. The results suggest that the incentive to default is clearly time-varying.

Figure 10 [about here]

By assumption, the country specific and market wide economic conditions explain the dynamics of the implied incentive to default. As such, I estimate the effect of stock market prices and the VIX on the individual conditional implied incentive to default over the 1998 - 2006 period, using the following model

\[
\left( \frac{c}{X} \right)_{i,t}^{\text{implied}} = \alpha_{1,i} + \alpha_{2,i} S_{i,t-1} + \alpha_{3,i} VIX_{t-1} + \varepsilon_t
\]

(21)

where \( \varepsilon_t \) denotes the heteroskedasticity consistent standard errors corrected with the Newey and West’s approach. The Table VI reports, for each country \( i \), the regression coefficients \( \hat{\alpha}_{1,i} \), \( \hat{\alpha}_{2,i} \) and \( \hat{\alpha}_{3,i} \), which are all statistically significant. The explanatory power \( (R^2) \) of the regression is on average 74%. These two factors can thus explain most of the time variation of the sovereign countries’ incentive to default. As a result, the implied default boundaries \( \left( \frac{c}{X} \right)_{i,t}^{\text{implied}} \) and the fitted ones \( \left( \frac{c}{X} \right)_{i,t}^{\text{fitted}} = \hat{\alpha}_{1,i} + \hat{\alpha}_{2,i} S_{i,t-1} + \hat{\alpha}_{3,i} VIX_{t-1} \) have a very close dynamics (see Figure 10), except for Russia.

The sovereign country’s incentive to default appears to be, on average, both country-specific and market-wide procyclical through the positive effect of stock market prices and the negative influence of the VIX. Indeed, the Table VI exhibits a contemporaneous correlation between the implied incentive to default \( \left( \frac{c}{X} \right)_{i,t}^{\text{fitted}} \) and the country-specific stock market prices \( S_{i,t} \) of 0.62, and a correlation with the market wide economic conditions \( VIX_t \) of -0.43. The conditional incentive to default is then lower than average in crisis periods, such as during the 1998’s Russian and 2003’s Brazilian default events, and higher than average in periods of economic growth. Hence, the conditional incentive to default generates interesting insights on the debate that contrasts the willingness to default with the incapacity to service debt. Because the incentive to default is low in periods of distress, the level of EMBI+ spreads suggests that sovereign countries experience difficulty in servicing their debt in such periods, eventually triggering default. However, credit spreads traded in the sovereign bond market imply an important incentive to default in presence of high economic growth, suggesting a willingness to service debt which is low in good times.
4.4 Out-of-sample Application: the 2007 - 2008 Subprime Crisis Period

This section provides an analysis of the dynamics of the predicted sovereign credit spreads during the 2007 - 2008 subprime crisis period. In contrast to the previous in-sample analyses, the credit spreads computed here do not incorporate future information in the estimation of the model parameters. I first generate the model credit spreads computed with the conditional incentive to default and then compare them with the observed EMBI+spreads. I also consider the model credit spreads computed under the assumption of a constant incentive to default and contrast the relative success of these two approaches.

Under the assumption of a conditional incentive to default, the credit spread \( CS_{TVB}^{Model,i,t} \) for country \( i \) at day \( t \) is determined by

\[
CS_{TVB}^{Model,i,t} = r \left[ 1 - \phi_i \left( \frac{r (\beta_i - 1) S_{i,t}/S_{i,0}}{\phi_i \beta_i} \left( \frac{c}{\lambda} \right)_{i,t}^{fitted} \right)^{\beta} \right]^{-1} - r
\]

with

\[
\left( \frac{c}{\lambda} \right)_{i,t}^{fitted} = \mathbb{E}_{t-1} \left[ \left( \frac{c}{\lambda} \right)_{i,t}^{implied} \mid \mathcal{F}_{t-1} \right] = \hat{\alpha}_{1,i,t} + \hat{\alpha}_{2,i,t} S_{i,t-1} + \hat{\alpha}_{3,i,t} VIX_{t-1}
\]

where the coefficient estimates \( \hat{\alpha}_{i,t} \) are computed using the increasing information set \( \mathcal{F}_{t-1} \), which incorporates information from January 1, 1998 through time \( t - 1 \). As displayed in Table VII, changes in the predicted credit spreads explain 22% of the time variation in EMBI+ spread changes over the 2007 - 2008 period, and up to 48% when including market-wide factors such as the VIX, the return on the S&P500 index, and the 5 year U.S. Treasury rate. These results provide clear evidence that both country-specific and global factors should be accounted for in the explanation of the predicted dynamics of sovereign credit spreads. This finding thus reconciles the in-sample results (see Section 4.2.3) with the empirical evidence provided by Longstaff et al. (2008) and Pan and Singleton (2008).

In Figure 11, I offer an out-of-sample comparison between the predicted credit spreads and those observed in the data. For space consideration, I only illustrate the results for Brazil and Russia. The model credit spreads \( CS_{TVB}^{Model,i,t} \) capture the levels and the time-variation in EMBI+ spreads during both the quiet period from 2007Q1 through 2008Q3 and during the sharp peaks and swings that occurred after the failure of Lehman Brothers in 2008Q4. Sovereign credit spreads were particularly affected during the second part of 2008. The rise in uncertainty in the U.S. economy, the deflationary prospects, and the slump in consumers’ confidence has triggered a fall in U.S. consumption over the last quarter of 2008. As a result, lower U.S.
imports coupled with decreasing commodity prices have negatively affected firms’ earnings in the emerging world. As the model predicts, lower fiscal revenues for governments have been accompanied by a severe rise in sovereign credit risk.

Figure 11 and Table VII [about here]

I now investigate the prediction of the model credit spreads computed under the assumption of a constant incentive to default, which is estimated over the in-sample period (1998 - 2006). As displayed in Table IV, changes in the predicted credit spread offer a good explanation of the time variation in EMBI+ spread changes. The quality of the explanation is even enhanced in the out-of-sample (see Analyses 2a, 2b, and 2c) compared to the in-sample period (see Analyses 1a, 1b, and 1c). The explanatory power corresponds to 17% and 32%, in- and out-of-sample respectively. In line with the results above, Analyses 2b and 2c also suggest that additional market-wide variables can significantly improve the out-of-sample predictions.

We can thus conclude that both approaches provide similar results when explaining credit spread changes. However, this is no longer the case when the model is used to explain credit spread levels. The difference is particularly striking in Figure 11. In Table V, I present a comparison of sample means and standard deviations for the daily predicted and EMBI+ spreads. Under the assumption of constant incentive to default, the predicted spreads underestimate the observed spreads of Mexico and Peru and overestimate the spreads of Brazil and Russia. Underestimation of credit spread has typically occurred in other studies. Due to the strong assumption of a constant incentive to default, this approach appears unsuccessful in the replication of the sample characteristics of daily EMBI+ spreads. However, under the assumption of a conditional incentive to default, Table V provides clear evidence that the model credit spreads present sample characteristics that are very close to those observed in the data, besides successfully explaining and predicting the time variation in EMBI+ spreads. In addition to the understanding of the mechanism behind a sovereign country’s decision to default, the importance of accounting for a time-varying incentive to default, which is affected by economic conditions, is then evident.

Table V [about here]

Finally, I analyze how the credit spread valuation with conditional incentive to default compares to the approach developed by Bodie et al. (2007). These authors were first to explain the dynamics of sovereign credit risk using market price data. In particular, they use weekly local-currency debt market prices measured in US dollars to infer the market value of the sovereign country’s assets and its volatility, and then...
compute weekly credit risk on foreign-currency debt. They analyze the distance-to-default measure\(^{27}\) for 11 developing countries in the period 2002 - 2004 and explain 80% of the time variation in the levels of credit spreads. As previously noted, an analysis on the levels of credit spreads can raise severe nonstationarity issues. However, for comparison purposes only, I follow Bodie et al. (2007) and compute the distance-to-default measure to explain the time variation in EMBI+ spreads over the out-of-sample period. The computation of this measure follows Bodie et al. (2007) but considers a conditional default boundary. The distance-to-default measure \(DD_{TVB}^{i,t}\) for country \(i\) at time \(t\), determined along the model’s assumptions of the current paper, is

\[
DD_{TVB}^{i,t} = \left( r - \frac{\sigma^2_i}{2} \right) T - \ln \left( \frac{v(i,t)}{s(i,t)} \right) \frac{\bar{\sigma}_i}{\sigma_i} \sqrt{T} 
\]

The time variation in the distance-to-default indicator captures 93% of the movements in the levels of daily EMBI+ credit spreads for Brazil, Mexico, Peru, and Russia (see Analyses 4a, 4b, and 4c in Table VII). Closely related to a sovereign country’s probability of defaulting, the distance-to-default measure appears to be a good indicator of that country’s creditworthiness. In addition, the inclusion of a conditional incentive to default enhances the results provided by the simple distance-to-default measure. Therefore, the approach developed in this paper seems to be crucial in the explanation of both the dynamics and the level of sovereign credit spreads.

5 Conclusion

Default should not be viewed as exogenous to a sovereign country’s decisions but rather as an optimal outcome. This paper develops a dynamic model of sovereign country default that determines the timing of default. The analysis also recognizes the role of strategic interaction between the sovereign country and its lenders. This paper merges both the theoretical and the empirical credit risk literature. One advantage of the model is that it offers an intuitive theoretical framework for understanding why and when a sovereign country defaults. Second, the model provides closed-form formulae for the recovery rate, credit spread, and

\(^{27}\)Following Moody’s KMV, the distance-to-default measure has been widely considered an indicator of credit risk (Campbell, Hilscher, and Szilagyi, 2007; Duffie, Saita, and Wang, 2007; and Bodie et al., 2007). This measure computes the difference between the implied forward market value of the sovereign country’s assets and the default boundary scaled by the standard deviation of the sovereign country’s assets. The distance-to-default measure \(DD_{i,t}\) for country \(i\) at time \(t\) thus captures how many standard deviations the sovereign country’s assets are away from default, which is defined by

\[
DD_{i,t} = \left( r - \frac{\sigma^2_i}{2} \right) T - \ln \left( \frac{v(i,t)}{s(i,t)} \right) \frac{\bar{\sigma}_i}{\sigma_i} \sqrt{T} 
\]
price of sovereign debt. Third, the theoretical predictions of the relationship between credit risk and other macroeconomic factors are consistent with the existing literature. This allows for diagnosis of movements in sovereign credit spreads through changes in these variables. Fourth, the estimated credit spreads can successfully capture the time variation in the changes of EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998-2008. Last, but not least, the paper shows that accounting for a time-varying incentive to default, which depends on both country-specific and market wide economic conditions, can successfully explain the sample characteristics of EMBI+ spreads. The framework developed in this paper thus lends itself to numerous pricing applications.
References


6 Appendix

6.1 Renegotiation Game

I present the renegotiation game between the sovereign country and its lenders and derive the recovery rate upon default $1 - \phi$. Sovereign debt $D(V)$ is priced in Section 2 using a default boundary $V^B$ that now accounts for the renegotiation process.

I first determine the allocation of the renegotiation surplus between a sovereign country and its lenders. The surplus represents the benefits of renegotiating the debt contracts compared to a full repudiation. For any debt reduction $\phi$, sovereign wealth upon default is

$$W(V)_{\mid V = V^B} = (1 - \lambda)V^B - \frac{c(1 - \phi)}{r}$$  \hfill (26)

In case of full repudiation ($\phi = 1$), the sovereign country saves $\frac{c(1 - \phi)}{r}$. However, direct sanctions are imposed by its lenders, magnifying the loss in sovereign wealth by a factor $k$. De Paoli et al. (2006) show that countries that do not restructure their debt face a median fall in economic activity that is several times larger than those which do. Under full repudiation, sovereign wealth satisfies

$$W(V)_{\mid V = V^B, \phi = 1} = V^B - k\lambda V^B$$  \hfill (27)

The difference between Eq. (23) and Eq. (24) represents the surplus on the sovereign side and equals

$$\Delta W(V)_{\mid V = V^B} = (k - 1)\lambda V^B - \frac{c(1 - \phi)}{r}$$  \hfill (28)

On the lender side, the recovered debt value at default $D(V)_{\mid V = V^B}$ is (see Eq. 4)

$$D(V)_{\mid V = V^B} = \frac{c(1 - \phi)}{r} = \Delta D(V)_{\mid V = V^B}$$  \hfill (29)

which determines the debt holders’ surplus $\Delta D(V)_{\mid V = V^B}$.

The expected value of debt is null if full repudiation applies ($D(V)_{\mid V = V^B, \phi = 1} = 0$). In Merton (1974), the value of the debt at default equals the residual value of the firm. I depart from this case, since there is no formal international bankruptcy court that would allow the debt holders to seize part of the existing assets. In addition, Bulow and Rogoff (1989) suggest that the sovereign assets that would be accessible to
creditors in the event of repudiation are negligible relative to the outstanding level of debt. Finally, parties would not be able to write a contract to guarantee a payment upon default. Nothing would then prevent the sovereign country from deviating at default, thus not honoring the contract.

I consider a Nash bargaining game to determine the recovered debt upon renegotiation. The sharing rule for the renegotiation surpluses upon default satisfies the Nash solution characterized as

\[
\phi^* = \arg \max_{0 \leq \phi \leq 1} \left[ \Delta W(V)|_{V=V_B} \right]^\eta \left[ \Delta D(V)|_{V=V_B} \right]^{1-\eta}
\]

\[
= \arg \max_{0 \leq \phi \leq 1} \left[ (k-1)\lambda V_B - \frac{c(1-\phi)}{r} \right]^\eta \left[ \frac{c(1-\phi)}{r} \right]^{1-\eta}
\]

\[\text{s.t. } \Delta W(V)|_{V=V_B} \geq 0\]

On one side, the sovereign country benefits from a smaller loss. The sanctions imposed in the case of full repudiation are avoided. However, it must continue to pay a fraction of the debt service. On the other side, debt holders benefit from receiving the recovered debt service. The outcome of the renegotiation process allocates both surpluses between the sovereign country and its lenders according to their bargaining power. I assume that there is a unique bargaining power for all lenders, which necessitates coordination between creditors, through the London or the Paris Clubs for example. This modeling assumption is in line with Hayri (2000), François (2006), and Yue (2006).

As long as both parties have some bargaining power \((0 < \eta < 1)\), there is a unique reduction \(\phi^*\) of the debt service that maximizes the surplus allocation between the two parties:

\[
\phi^* = 1 - \frac{r\lambda V_B(k-1)(1-\eta)}{c} \geq \bar{\phi}
\]

The solution of \(\phi^*\) does not have a distribution support between 0 and 1, at least given the assumed specification. There is a lower bound \(\bar{\phi}\) in the debt reduction, below which renegotiation is never observed. For the sovereign country to be willing to renegotiate, the costs of bearing the new debt service must not outweigh the benefits of reduced economic sanctions. The lower threshold \(\bar{\phi} = \{\phi : \Delta W(V)|_{V=V_B} = 0\}\) denotes the point of indifference between renegotiating or not.
6.1.1 Credit Spread with Endogenous Recovery Rate

Accounting for the endogenous recovery rate derived above, the credit spread is

\[ CS^R \equiv \frac{c}{D(V, c)} - r \]  
\[ = r \left[ 1 - \phi^* \left( \frac{V}{V^{R*}} \right)^\beta \right]^{-1} - r \]

where the default boundary \( V^{R*} \) maximizes \( W(V) \), along the usual smooth-pasting condition, and is defined by

\[ V^{R*} = \frac{c \beta}{r \lambda \left( \beta (1 + (k - 1)(1 - \eta)) - 1 \right)} \]

and the debt reduction upon default is

\[ \phi^* = 1 - \frac{V^{R*}(k - 1)(1 - \eta) r \lambda}{c} \]
\[ = 1 - \frac{(k - 1)(1 - \eta) \beta i}{\beta (1 + (k - 1)(1 - \eta)) - 1} \]

with

\[ \beta = \frac{1}{2} - \frac{r}{\sigma^2} - \sqrt{\left( \frac{1}{2} - \frac{r}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0 \]
6.2 Figures

Figure 1: Credit Risk and Related Macro-Variables. The figure illustrates the predicted relationships between the macro-variables considered in the model and the model credit spreads. Each panel compares the credit spread computed with two different models: the benchmark model and the model accounting for endogenous renegotiation upon default. The benchmark model credit spread is computed under the assumption of either a fixed or an exponential default boundary. All spreads are computed with the parameter values of Table I.
Figure 2: Recovery Rate and Related Macro-variables. The figure illustrates the recovery rate determined by the Nash bargaining game in function of a set of variables provided by the model. The spreads are computed under the assumption of a fixed default boundary with the parameter values of Table I.
Figure 3: Stock Market Index Prices and EMBI+ Spreads, 1998 - 2008. The figure plots the dynamics of the daily EMBI+ spreads and the daily stock market index price levels (normalized to unity at the start of the period) for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008.
Figure 4: Relationship between Stock Market Index Prices and EMBI+ Spreads, 1998 - 2008. The figure illustrates the relationship between country stock market index prices (normalized to unity at the start of the period) and EMBI+ spreads. The sample consists of daily data for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008.
Figure 5: Dynamic Correlation between Model Credit Spreads, VIX, and EMBI+ Spreads, 1998 - 2008. The figure displays the dynamics of the correlation between EMBI+ spreads and the model credit spreads (black solid line), and between EMBI+ spreads and the option-implied volatility index VIX (blue dotted line) for Brazil, Mexico, Peru, and Russia over the period 1998 - 2008. The time-varying correlation is computed with the Dynamic Conditional Correlation (DCC) Multivariate GARCH model developed by Engle and Sheppard (2001). The spreads are computed with the parameter values presented in Table II.
Figure 6: EMBI+ Spreads and Model Credit Spreads Computed with Stock Market Prices, 1998 - 2008. The figure offers a comparison between the model credit spreads and the EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the in-sample period 1998 - 2006. The model credit spreads are computed using stock market indices measured in U.S. Dollars and the parameter values presented in Table II.
Figure 7: Changes in EMBI+ Spreads and Model Credit Spreads Computed with Stock Market Prices, 1998 - 2006. The figure compares the changes in model credit spreads with the changes in EMBI+ spreads for Brazil and Russia over the period 1998 - 2006. The daily credit spreads are computed using stock market indices measured in U.S. Dollars and the parameter values presented in Table II.
Figure 8: EMBI+ Spreads and Model Credit Spreads Computed with GDP Data, 1998 - 2006. The figure compares the model credit spreads with EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998 - 2006. The quarterly credit spreads are computed using GDP data in U.S. Dollars and the parameter values presented in Table II.
Figure 9: Changes in EMBI+ Spreads and Model Credit Spreads Computed with GDP Data, 1998 - 2006. The figure compares changes in the model credit spreads with changes in EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the period 1998 - 2006. The quarterly credit spreads are computed using GDP data in U.S. Dollars and the parameter values presented in Table II.
Figure 10: Implied and Fitted Time-varying Incentive to Default, 1998 - 2006. The figure shows the implied incentive to default for Brazil, Mexico, Peru, and Russia over the period 1998 - 2006 computed as

\[ \text{i}_{t,t}^{\text{implied}} = \frac{V_{t,t}}{V_{t,t}^0} \left( \frac{C_{SEMBIL,t}}{C_{SEMBIL,t} + r} \right)^{\phi_i} - \beta_i \]

and compares them with the fitted series, determined by

\[ \text{i}_{t,t}^{\text{fitted}} = \hat{\alpha}_1 \text{i}_{t-1} + \hat{\alpha}_2 \text{i}_{t-1} + \hat{\alpha}_3 V\text{IX}_{t-1} \]

The coefficient estimates are presented in Table VII and the remaining parameter values in Table II.
Figure 11: Out-of-sample Predicted Credit Spreads, 2007 - 2008. The figure illustrates the EMBI+ spreads and the predicted model credit spreads for Brazil and Russia over the out-of-sample period 2007 - 2008, computed with a constant (left panels) and a conditional (right panels) incentive to default. The conditional incentive to default is determined by \(\hat{\chi}_{t,t}^{fitted} = \hat{\alpha}_1 + \hat{\alpha}_2 S_{i,t-1} + \hat{\alpha}_3 VIX_t\).
### Table I: Parameter Choices

This table presents the parameter values considered in the theoretical predictions on the relationships between the model credit spread, the recovery rate, and the macroeconomic fundamentals.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Range Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Level of Assets $V$</td>
<td>100.0</td>
<td>[50 : 150]</td>
<td></td>
</tr>
<tr>
<td>Assets’ Volatility $\sigma$</td>
<td>0.300</td>
<td>[0 : 0.50]</td>
<td>Average stock market volatility (1998-2006)</td>
</tr>
<tr>
<td>Costs of Defaulting $\lambda$</td>
<td>0.07</td>
<td>[0.06 : 0.16]</td>
<td>Author’s assumption</td>
</tr>
<tr>
<td>Risk-Free Rate $r$</td>
<td>0.056</td>
<td>[0.02 : 0.10]</td>
<td>Average 5Y US Treasury rate (1998-2006)</td>
</tr>
<tr>
<td>Debt Loss upon Default $\phi$</td>
<td>0.660</td>
<td>[0 : 1]</td>
<td>Average Moody’s estimate (1983-2005)</td>
</tr>
<tr>
<td>Sovereign Bargaining Power $\eta$</td>
<td>0.750</td>
<td>[0 : 1]</td>
<td>Author’s assumption (0.5 in François (2005) and 0.83 in Yue (2006))</td>
</tr>
<tr>
<td>Non-Renegotiation Sanctions $k$</td>
<td>5.000</td>
<td></td>
<td>De Paoli, Hoggarth, &amp; Saporta (2006)</td>
</tr>
</tbody>
</table>
Table II: Estimated Parameters. This table presents the parameter values estimated from the equation \( \{(\hat{\phi}, \hat{\phi}_i, \hat{\sigma}_i) = \arg\min(\sum_t (\Delta CSEMBI_{i,t} - \Delta CS_{Model_{i,t}})^2) s.t. \phi_i < 1 \) over the in-sample period 1998 - 2006. CS_{EMBI} and CS_{Model} stand for the observed EMBI+ spreads and the credit spreads estimated by the model, respectively, for country \( i \) at time \( t \).

### ESTIMATED PARAMETER VALUES

<table>
<thead>
<tr>
<th>Recovery Rate</th>
<th>Incentive to Default</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 - \phi )</td>
<td>( \hat{\phi} )</td>
<td>( \hat{\sigma} )</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.46</td>
<td>0.11</td>
</tr>
<tr>
<td>Peru</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Russia</td>
<td>0.18</td>
<td>1.79</td>
</tr>
<tr>
<td>Average</td>
<td>0.36</td>
<td>0.58</td>
</tr>
</tbody>
</table>

**Credit Spreads Computed with Stock Market Prices: Daily Frequency**

| Brazil        | 0.41                 | 0.15       | 0.36       |
| Mexico        | 0.49                 | 0.12       | 0.23       |
| Peru          | 0.46                 | 0.12       | 0.24       |
| Russia        | 0.31                 | 0.19       | 0.47       |
| Average       | 0.42                 | 0.15       | 0.33       |

**Credit Spreads Computed with GDP Data: Quarterly Frequency**
Table III: Correlation between Model and Observed Credit Spreads. The table provides the Pearson correlation coefficients between the model credit spreads, computed with Eq. 15, and the EMBI+ spreads, in levels and in changes. The table breaks down the in-sample period 1998 - 2006 and the out-of-sample period 2006 - 2008. The credit spreads are computed using stock market prices in U.S. Dollars at the daily frequency and GDP data at the quarterly frequency.

<table>
<thead>
<tr>
<th>Credit Spreads Computed with Stock Market Prices: Daily Frequency</th>
<th>Correlation in levels</th>
<th>Correlation in changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>0.93</td>
<td>0.62</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>Peru</td>
<td>0.89</td>
<td>0.78</td>
</tr>
<tr>
<td>Russia</td>
<td>0.89</td>
<td>0.93</td>
</tr>
<tr>
<td>Average</td>
<td>0.90</td>
<td>0.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Credit Spreads Computed with GDP Data: Quarterly Frequency</th>
<th>Correlation in levels</th>
<th>Correlation in changes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.28</td>
<td>0.71</td>
</tr>
<tr>
<td>Peru</td>
<td>0.79</td>
<td>0.17</td>
</tr>
<tr>
<td>Russia</td>
<td>0.96</td>
<td>0.80</td>
</tr>
<tr>
<td>Average</td>
<td>0.72</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Table IV: Estimation of EMBI+ Spreads with Daily Model Credit Spreads. This table presents GLS regression results from a random effect model, using a panel data with daily observations. The out-of-sample analysis consists of the period from January 1, 1998 through December 31, 2006 and the out-of-sample analysis covers the period from January 1, 2007 through December 31, 2008. The benchmark regression is $\Delta CS_{EMBI,i,t} = \delta_i \Delta CS_{Model,i,t} + \omega_i + \nu_{i,t}$. The heteroskedasticity consistent standard errors reported in parentheses are corrected for serial correlation and cross-sectional dependence using the Driscoll and Kraay’s (1998) extension of Newey and West’s non-parametric variance covariance estimator (*, **, *** relate to coefficients significant at the 90, 95, 99% confidence level, respectively.)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>EMBI+ Spread $\Delta CS_{EMBI,i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis</td>
<td>1a 1b 1c 2a 2b 2c</td>
</tr>
<tr>
<td>Constant</td>
<td>0.000 0.000 0.000 0.000 0.000 0.000</td>
</tr>
<tr>
<td>Model Credit Spread $\Delta CS_{Model,i,t}$</td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td>1.000*** 0.939*** 0.941*** 0.851*** 0.631*** 0.611***</td>
</tr>
<tr>
<td>Mexico</td>
<td>1.002*** 0.790*** 0.787*** 3.318*** 2.363*** 2.315***</td>
</tr>
<tr>
<td>Peru</td>
<td>1.000*** 0.834*** 0.849*** 4.254*** 2.741*** 2.613***</td>
</tr>
<tr>
<td>Russia</td>
<td>1.001*** 0.975*** 0.972*** 0.873*** 0.773*** 0.731***</td>
</tr>
<tr>
<td>Option-implied Volatility $\Delta VIX_t$</td>
<td>0.033*** 0.046*** 0.011*** 0.012***</td>
</tr>
<tr>
<td>S&amp;P 500 Return $R_{SP500_t}$</td>
<td></td>
</tr>
<tr>
<td>0.024*** 0.011***</td>
<td></td>
</tr>
<tr>
<td>5Y US Treasury Rate $\Delta UST_t$</td>
<td>-0.427*** -0.581***</td>
</tr>
<tr>
<td>$R^2_{Within}$</td>
<td>0.172 0.180 0.184 0.320 0.353 0.454</td>
</tr>
<tr>
<td>$R^2_{Between}$</td>
<td>0.929 0.926 0.926 0.600 0.668 0.659</td>
</tr>
<tr>
<td>$R^2_{Overall}$</td>
<td>0.172 0.187 0.184 0.320 0.353 0.454</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>9384 9384 9384 2092 2092 2092</td>
</tr>
</tbody>
</table>

This table provides descriptive statistics for the model credit spreads and EMBI+ spreads for Brazil, Mexico, Peru, and Russia over the out-of-sample period 2006 - 2008. The table also offers a comparison of the model credit spreads assuming a constant incentive to default, as computed in Eq. (15), and the model credit spreads. The model credit spreads are computed with a time-varying incentive to default, given by $\left( \frac{z}{\hat{z}} \right)_{i,t}^{fitted} = \hat{\alpha}_1,i + \hat{\alpha}_2,i S_{i,t-1} + \hat{\alpha}_3,i VIX_{t-1}$, where the coefficients estimates are presented in Table VII.

<table>
<thead>
<tr>
<th></th>
<th>Constant Incentive to Default</th>
<th>Time-varying Incentive to Default</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (Standard Deviation)</td>
<td>Mean (Standard Deviation)</td>
</tr>
<tr>
<td>Brazil</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model spreads</td>
<td>0.0531 (0.1634)</td>
<td>0.0249 (0.1936)</td>
</tr>
<tr>
<td>EMBI+ spreads</td>
<td>0.0241 (0.2317)</td>
<td>0.0241 (0.2317)</td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model spreads</td>
<td>0.0072 (0.0390)</td>
<td>0.0160 (0.2250)</td>
</tr>
<tr>
<td>EMBI+ spreads</td>
<td>0.0168 (0.2348)</td>
<td>0.0168 (0.2348)</td>
</tr>
<tr>
<td>Peru</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model spreads</td>
<td>0.0042 (0.0241)</td>
<td>0.0194 (0.2377)</td>
</tr>
<tr>
<td>EMBI+ spreads</td>
<td>0.0206 (0.2783)</td>
<td>0.0206 (0.2783)</td>
</tr>
<tr>
<td>Russia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model spreads</td>
<td>0.1636 (0.2782)</td>
<td>0.0192 (0.3424)</td>
</tr>
<tr>
<td>EMBI+ spreads</td>
<td>0.0208 (0.4200)</td>
<td>0.0208 (0.4200)</td>
</tr>
</tbody>
</table>
Table VI: Correlations and Parameter Estimates of the Time-varying Incentive to Default, 1998 - 2006. The table gives the Pearson correlations between the implied incentive to default, the stock market index level $S$, and the VIX. The implied incentive to default for country $i$ at time $t$ is computed as 
$$(\frac{\lambda_i}{\lambda})_{\text{implied},i,t} = \frac{V_{i,t}r}{V_{i,0}\phi_{i,t}} \left( \frac{CS_{EMBI,i,t}}{CS_{EMBI,i,t}+r} \right) \phi_i$$
where $V_{i,t}$ is the stock market index level for country $i$, $C_{EMBI,i,t}$ is the country's EMBI index at time $t$, $r$ is the risk-free rate, and $\phi_i$ is a factor load. The table also provides the coefficient estimates of the regression $$\left(\frac{\lambda_i}{\lambda}\right)_{\text{implied},i,t} = \alpha_{1,i} + \alpha_{2,i}S_{i,t-1} + \alpha_{3,i}VIX_{t-1} + \varepsilon_t$$ over the period 1998 - 2006. The heteroskedasticity consistent standard errors are corrected for serial correlation using the Newey and West’s non-parametric variance covariance estimator (*, **, *** relate to coefficients significant at the 90, 95, 99% confidence level, respectively.)

<table>
<thead>
<tr>
<th>Stock</th>
<th>Correlation with the Implied Incentive to Default</th>
<th>OLS Coefficient Estimates of $$(\frac{\lambda}{\lambda})<em>{\text{implied},i,t} = \alpha</em>{1,i} + \alpha_{2,i}S_{i,t-1} + \alpha_{3,i}VIX_{t-1} + \varepsilon_t$$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>Stock Market Index Price</td>
<td>VIX</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.97</td>
<td>-0.66</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.97</td>
<td>-0.66</td>
</tr>
<tr>
<td>Peru</td>
<td>0.99</td>
<td>-0.71</td>
</tr>
<tr>
<td>Russia</td>
<td>-0.47</td>
<td>0.29</td>
</tr>
<tr>
<td>Average</td>
<td>0.62</td>
<td>-0.43</td>
</tr>
</tbody>
</table>
Table VII: Alternative Estimations of EMBI+ Spreads. This table presents GLS regression results from a random effect model, using a panel data with daily observations. The out-of-sample analysis consists of the period from January 1, 1998 through December 31, 2006 and the out-of-sample analysis covers the period from January 1, 2007 through December 31, 2008. Analyses 3a, 3b, and 3c consider the regression $\Delta CS_{EMBI,t} = \delta_i \Delta CS_{TVB Model,i,t} + \omega_i + \nu_{i,t}$, where $CS_{TVB Model,i,t}$ denotes the credit spread computed with a time-varying incentive to default. Analyses 4a, 4b, and 4c consider the regression $CS_{EMBI,i,t} = \delta_2 DD_{Model,i,t} + \omega_i + \nu_{i,t}$, where $DD_{Model,i,t}$ is the distant-to-default measure. The heteroskedasticity consistent standard errors reported in parentheses are corrected for serial correlation and cross-sectional dependence using the Driscoll and Kraay’s (1998) extension of Newey and West’s non-parametric variance covariance estimator (*, **, *** relate to coefficients significant at the 90, 95, 99% confidence level, respectively.)

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>$\Delta$EMBI+ Spread</th>
<th>$\Delta CS_{EMBI,i,t}$</th>
<th>$\Delta$Model Credit Spread</th>
<th>$CS_{EMBI,i,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Explanatory Variable</td>
<td>$\Delta$EMBI+ Spread</td>
<td>$\Delta CS_{EMBI,i,t}$</td>
<td>$\Delta CS_{TVB Model,i,t}$</td>
<td>$\Delta$Model Credit Spread</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.187***</td>
<td>0.107***</td>
<td>0.104***</td>
<td>-0.058***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.164***</td>
<td>0.111***</td>
<td>0.111***</td>
<td>-0.047***</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.013)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Peru</td>
<td>0.057***</td>
<td>0.017**</td>
<td>0.017**</td>
<td>-0.085***</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.010)</td>
<td>(0.09)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Russia</td>
<td>0.292***</td>
<td>0.268***</td>
<td>0.267***</td>
<td>-0.041***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>Global Factors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Option-implied Volatility $\Delta VIX_t$</td>
<td>0.018***</td>
<td>0.016***</td>
<td>-0.012***</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>S&amp;P 500 Return $R_{SP500}$</td>
<td>0.009***</td>
<td>0.028***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>US Treasury Rate $\Delta UST_t$</td>
<td>-0.612***</td>
<td>-0.227*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.131)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2_{Within}$</td>
<td>0.222</td>
<td>0.333</td>
<td>0.447</td>
<td>0.884</td>
</tr>
<tr>
<td>$R^2_{Between}$</td>
<td>0.692</td>
<td>0.757</td>
<td>0.760</td>
<td>0.036</td>
</tr>
<tr>
<td>$R^2_{Overall}$</td>
<td>0.222</td>
<td>0.333</td>
<td>0.447</td>
<td>0.291</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>2092</td>
<td>2092</td>
<td>2092</td>
<td>2092</td>
</tr>
</tbody>
</table>